



Article

Motivational Stage of Vector Method Training

Rakhimzhanov Jamshid Khayotovich^{*1}

1. Master's student of 2nd year of JSPU in the direction of "Methodology of teaching exact and natural sciences" (mathematics)

* Correspondence: 200209jr@gmail.com

Abstract: The application of vector methods in solving mathematical problems represents one of the effective pedagogical approaches aimed at enhancing student motivation. This methodology not only helps students master vector formulas and apply them to specialized problems but also develops their ability to utilize vector techniques in solving other types of mathematical problems. This approach contributes to: increasing students' cognitive interest in mathematics; developing the skill of applying vector methods across diverse mathematical contexts; fostering a deeper understanding of geometric and algebraic relationships. Thus, integrating vector methods into the learning process enhances not only computational skills but also promotes flexible mathematical reasoning.

Keywords: Vectors, Scalar Multiplication, Vector Multiplication, Eliminating Uncertainty, Collinear Vectors

1. Introduction

Students often encounter greater difficulties when studying vectors in the school mathematics curriculum compared to other topics [1]. One of the primary reasons for this challenge is the lack of student interest and motivation in learning this subject [2]. Therefore, to enhance students' understanding of vectors, it is essential to first cultivate their engagement with the topic. Several pedagogical approaches can be employed to achieve this objective [3], [4].

In my view, an effective strategy involves demonstrating how vector methods can simplify the solution of complex problems. This can be illustrated through a practical example, showing students how vectors provide an efficient and logical approach to problem-solving [5]. At this stage, simple and easy examples are primarily provided to students, which contribute to a deeper understanding and mastery of the material. Here are the main reasons why this is important.

2. Materials and Methods

The figure shows two straight lines; using this graph, find the angle between them. Before solving the problem, let us consider Figure 1, which visually represents two intersecting straight lines. The purpose of this figure is to provide a geometric understanding of the problem at hand: determining the angle between the lines using both traditional and vector methods.

These points form the basis for constructing equations and direction vectors.

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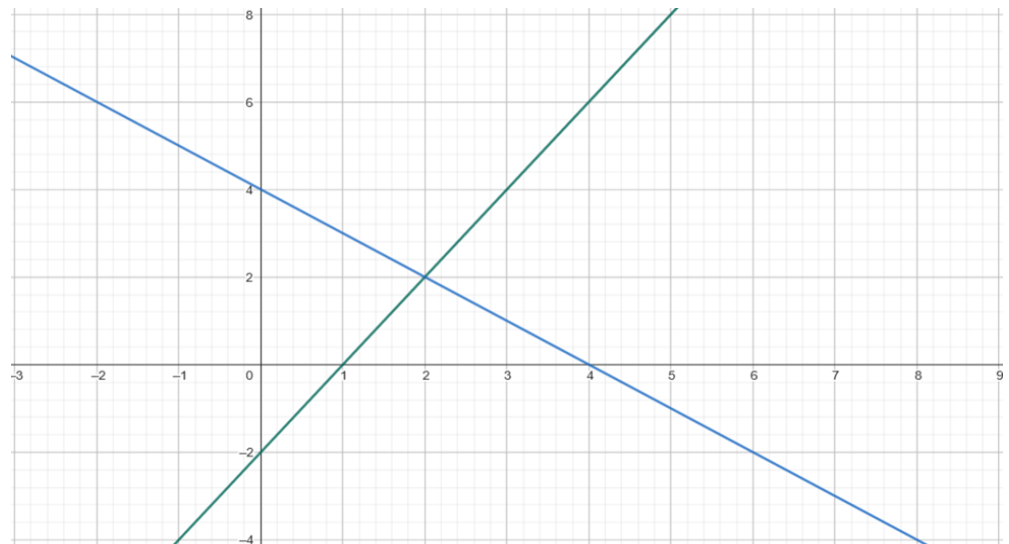


Figure 1. The Angle Between Two Intersecting Lines is Represented Graphically.

As is commonly known, determining the angle between two straight lines requires first establishing their equations. Subsequently, one must employ the tangent formula for calculating the angle between lines $\text{tga} = \left| \frac{k_2 - k_1}{1 + k_1 * k_2} \right|$ [6].

Solving this example using the given method may require considerable time and effort, and there is also the possibility of making mechanical errors when deriving the equations of the lines. The solution will take the following form;

Consider three points: point A(2,2) as the intersection of the lines, with points B(4,6) and C(4,0) lying on the green and blue lines respectively. Given two points A(x_1, y_1) and B(x_2, y_2) on the green line, the equation of this straight line can be determined using the two-point form equation: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ [7].

For points A(4,6) and B(2,2), the equation derivation proceeds as follows:

$$\frac{y - 6}{2 - 6} = \frac{x - 4}{2 - 4} \leftrightarrow \frac{y - 6}{-4} = \frac{x - 4}{-2}$$

Simplification: $\frac{y - 6}{2} = \frac{x - 4}{1}$ (multiplying both sides by -2)

$$y - 6 = 2(x - 4) \leftrightarrow y = 2x - 2 \text{ (Final equation of the line).}$$

To determine the equation of the blue line, let us consider two points lying on this line: B(x_1, y_1) and C(x_2, y_2). For our specific case, we take B(2,2) and C(4,0). The line equation passing through these points can be found using the same two-point form formula.

$$\frac{y - 2}{0 - 2} = \frac{x - 2}{4 - 2} \leftrightarrow y = -x + 4.$$

After deriving the equations of both lines, we can calculate the angle between them using the tangent formula for intersecting lines:

$$\text{tga} = \left| \frac{k_2 - k_1}{1 + k_1 * k_2} \right|.$$

k_1 = slope of the green line (previously found: $k_1 = 2$)

k_2 = slope of the blue line (previously found: $k_2 = -1$)

$$\text{tga} = |(-1 - 2)/(1 - 1 * 2)| \leftrightarrow \text{tga} = 3,$$

To determine the exact angle value from the tangent ratio, we apply the inverse trigonometric function: $\alpha = \arctg 3 \leftrightarrow \alpha = 71,57^\circ$.

As demonstrated, determining the angle between lines traditionally requires students to apply multiple formulas and computational steps. However, an alternative

vector-based approach offers a more streamlined solution through the dot product formula [7].

Given two vectors in the plane, $\vec{a}(x_1; y_1)$ and $\vec{b}(x_2; y_2)$, there exist two fundamental types of vector product. First one is scalar product with formula $\vec{a} * \vec{b} = |\vec{a}| * |\vec{b}| * \cos\alpha$, ($|\vec{a}| = \sqrt{x_1^2 + y_1^2}$; $x_1, y_1 \in R^2$) [8] or by only coordinate representation: $\vec{a} * \vec{b} = x_1x_2 + y_1y_2$ [5]. Second one is vector product, which is described by formula of a $\vec{a} \times (b) = c$. To find the angle between two vectors, we will use the formula derived from the dot product. By equating these two expressions of the dot product and dividing both sides of the equation by $\|a\| \cdot \|b\|$, we obtain the following formula:

$$\cos\alpha = \frac{\vec{a} * \vec{b}}{|\vec{a}| * |\vec{b}|} = \frac{|x_1x_2 + y_1y_2|}{\sqrt{x_1^2 + y_1^2} * \sqrt{x_2^2 + y_2^2}} \quad [9]$$

We now proceed to solve this example using the vector method. Following our previous approach with linear equations, we will utilize the same set of points on the green line: A(2;2) and B(4;6). Given two points on a straight line, we can construct a direction vector. Let us define this as vector \vec{a} :

$$\vec{a} = \overrightarrow{AB} (4 - 2; 6 - 2) = (2; 4)$$

For the second vector, we again utilize the previously defined points on the blue line: B(2;2) and C(4;0). By converting these points into a directional vector, we obtain:

$$\vec{b} = \overrightarrow{BC} (4 - 2; 0 - 2) = (2; -2)$$

The next phase of our work involves determining the cosine of the angle ($\cos\alpha$) between the two vectors. For this purpose, we will employ the formula previously derived:

$$\cos\alpha = \frac{\vec{a} * \vec{b}}{|\vec{a}| * |\vec{b}|} = \frac{|x_1x_2 + y_1y_2|}{\sqrt{x_1^2 + y_1^2} * \sqrt{x_2^2 + y_2^2}} \quad [10]$$

$$\vec{a} = \overrightarrow{AB} (2; 4) \text{ and } \vec{b} = \overrightarrow{BC} (2; -2)$$

$$\cos\alpha = \frac{\vec{a} * \vec{b}}{|\vec{a}| * |\vec{b}|} = \frac{|2*2 + 4*(-2)|}{\sqrt{2^2 + 4^2} * \sqrt{2^2 + (-2)^2}} \leftrightarrow \cos\alpha = \frac{|-4|}{\sqrt{20} * \sqrt{8}} \text{ or } \cos\alpha = \frac{|-1|}{\sqrt{10}}$$

To determine the exact angle value from the tangent ratio, we apply the inverse trigonometric function: $\alpha = \arctan 3 \leftrightarrow \alpha = 71,57^\circ$.

3. Results and Discussion

As we can observe, both solution methods for this problem yield identical results, yet they differ substantially in their complexity levels.

Traditional Slope Method requires: deriving line equations from two points, applying proportionality principles, calculating tangent of slope angles, performing multiple algebraic transformations, notable drawbacks [11], [12] time-intensive computations, high probability of calculation errors, multiple conceptual prerequisites [13].

Vector Approach demonstrates superior efficiency through: direct vector construction from coordinates, application of a single dot product formula, straightforward magnitude computation [14]. Key advantages: streamlined computational process, reduced error susceptibility, enhanced geometric intuition [15].

4. Conclusion

As we see the solution of this example with two ways gives the same value, but the ways to solve the problem has a different level of complexity. As previously said, solving the problem with the first way requires knowledge of several topics (formula equation passing through two points, proportion, tangent of angle coefficient, etc.) which will take a very long time and can lead to mechanical errors. But by using the vector method these problems are much simplified and can increase the student's level of motivation to learn the topic of vectors.

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